

Noise reduction in Interferometric Gravitational Wave Detectors

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Abstract

Gravitational waves have now been detected multiple times by interferometric gravitational wave detectors around the world such as LIGO and VIRGO - a century after being first predicted by Dr. Albert Einstein. Here, I present a report in which I review the nature of gravitational waves, the working of interferometric GW detectors, the various noises plaguing them, and techniques to reduce the different noises.

1. Gravitational Waves

According to the general theory of relativity published by Dr. Albert Einstein in 1915, we live in a 4-dimensional universe which has 3 dimensions of space and 1-dimension of time. Together, these 4-dimensions, also called *spacetime*, form the fabric of the universe. This so-called spacetime is not passive in nature, but rather active. It can bend, warp and theoretically tear up. According to the field equation (given below) of general relativity, mass and energy deform or warp this spacetime by their presence and this warpage is manifested physically as an attractive force that we know as gravity.

The left-hand side of the equation describes the warpage or curvature of spacetime produced by the presence of matter or energy and on the right-hand side, the term $T_{\mu\nu}$ called the *energy-momentum tensor*, describes the mass-energy distribution that is deforming the spacetime.

This simple looking equation or set of equations sometimes makes bizarre predictions. One such prediction is the existence of gravitational waves. These are the waves in the very fabric of the universe-*spacetime*. These transverse and traceless waves originate from changing gravitational field and travel at the speed of light hence following the principle of causality of special relativity. Such changes in the gravitational field in a localized space are only possible through accelerating matter or energy or both. If we assume that an observer is very distant from a matter distribution that is at rest, the spacetime around him/her is described by the metric $\square_{\mu\nu}$. Whenever there is a change in the term $T_{\mu\nu}$ or the mass distribution due to some accelerated motion, the initial metric will be altered to take the following form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}.$$

$$\check{g}_{\mu\nu} = \square_{\mu\nu} + h_{\mu\nu}$$

This new tensor introduced in the equation and obtained by solving Einstein’s equations, $h_{\mu\nu}$, describes the propagation of the warpage in spacetime or the gravitational waves so formed. These gravitational waves have been currently detected only by interferometric detectors and hence we review the working of such detectors first place in the coming section.

2. Operation of Interferometric Gravitational Wave Detectors

All the interferometric gravitational wave detectors around the world such as LIGO, VIRGO, GEO 600, etc. exploit one crucial property of passing gravitational waves-*tracelessness*. This implies that whenever gravitational waves traverse perpendicular to the plane of an interferometer, they stretch one arm and contract the other and after some time do the opposite. This stretching and contracting of the arms or basically spacetime could be sensed by the change in light travel time between masses kept along orthogonal arms. This change in light travel time can be measured by an extremely sensitive L-shaped Michelson interferometer as gravitational waves are quadrupolar in nature.

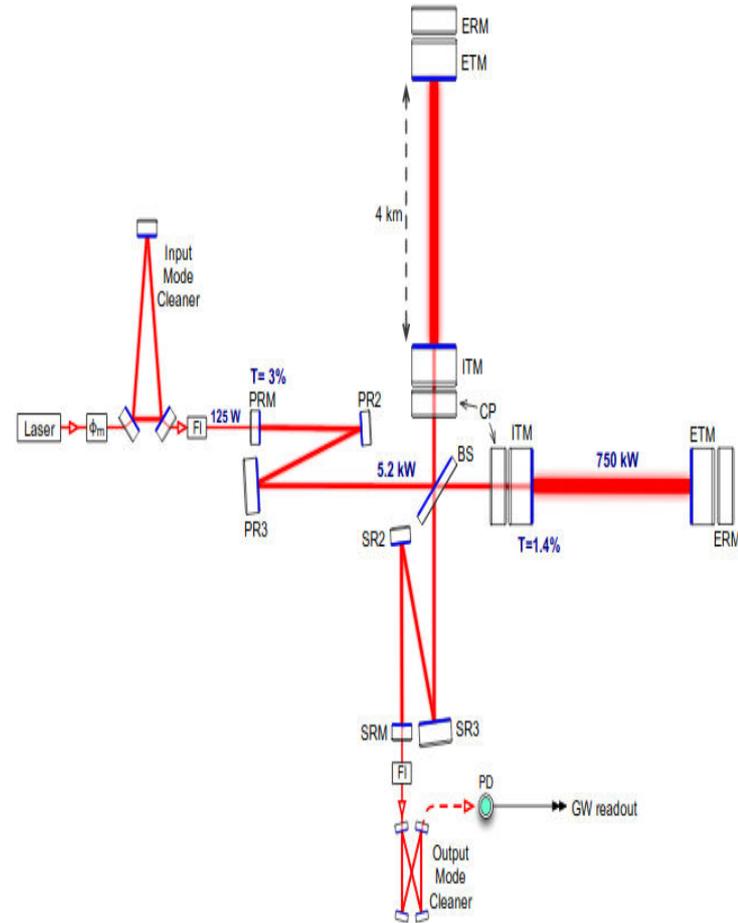


Image courtesy of aLIGO-P1400177-v5: <https://arxiv.org/pdf/1411.4547.pdf>

As depicted above, a laser beam travels along two orthogonal paths of the interferometer after being split by a beam splitter (BS) and combines to form an interference pattern at the photodetector (PD). If there is a change in the relative lengths of the two paths, there occurs a phase difference in the fractions of the laser in the arms resulting in an alteration in the interference pattern from which data is extracted about a passing gravitational wave. However, the lengths of the two arms are so adjusted that they are precisely equal so that the lasers in the arms interfere destructively towards the PD and are instead redirected towards the laser source. Hence, no

interference pattern appears on the detector. Only when a gravitational wave passes does the detector measure an interference pattern.

Modern laser GW interferometers are specialized Michelson interferometers with multiple optical cavities like Fabry-Perot resonant cavities to increase light interaction time with a gravitational wave as described by the following equation:

$$\Delta\phi(t) = h(t)\tau_{rt0}\frac{2\pi c}{\lambda}$$

The above equation computes the phase difference of the two lasers in the orthogonal arms as a fraction h of the total phase that the laser accumulates as it traverses the interferometer. Thus, if we let light travel for a longer time, there would be a greater phase difference resulting in increased sensitivity. There is a plethora of other instrumentation in the apparatus such as power recycling mirrors (PRM), signal recycling mirrors (SRM), phase modulator (ϕ_m), Faraday isolator (FI), compensation plates (CP), test masses, etc. which help advanced gravitational-wave interferometers attain the sensitivity levels at the orders of magnitude of about 10^{-21} . However, with increased sensitivity, the interferometer is prone to many minute noises that defy the detectors.

3. Noise Sources and mitigation techniques

Interferometric gravitational wave detectors' readings are inhabited by tons of noises such as laser frequency noise, photodetector dark noise, actuator noise, thermal noise, quantum noise, etc. which, if not filtered, might give wrong signals to the system of having detected a gravitational wave. To filter out such noises, firstly they must be removed to a large extent via precision manufacturing and configuring of the instruments involved and appropriate isolation. Then the remnant noises could be further identified and filtered by using statistical methods for instance matched filtering and χ^2 veto. Thus, the identification of noise sources becomes ever more important. Major types of noises include:

1. Photon-Shot Noise: All interferometric gravitational wave detectors operate at the dark fringe i.e. no light falls on the PD until a GW passes as discussed above. However, due to the fundamentals of quantum theory, the arrival of individual photons of light is an independent event and hence random. Thus, this results in random fluctuations in the laser's intensity that might result in a shift from the dark fringe condition and hence light appears on the PD. If we don't account for such random

laser intensity fluctuations, we will falsely conclude that we detected a GW signal. Now, since it happens randomly, the error introduced by photon-shot noise improves with the increase in the number of photons used in the laser.

$$h_{shot}(f) = \frac{1}{L} \sqrt{\frac{\hbar c \lambda}{2\pi P_{in}}}$$

The above equation relating the amplitude of the shot-noise and the laser power clearly gives us the solution. If we increase the laser power that is input in the interferometer, photon shot-noise could be minimized to a large extent. One obvious measure would be just to use a more powerful laser. However, one less obvious method is the recycling of the laser that returns towards the laser source from the beam splitter due to the dark fringe condition. This is why power recycling mirrors (PRM) are used between the beam splitter and the laser source. This cavity redirects the returning laser in such a precise manner that when exiting it is exactly in phase with the laser input from the source itself resulting in the reinforcement of the laser power. Since a very tiny amount of laser energy is lost in the highly polished

mirrors, the laser power gradually builds up countering the shot-noise.

2. Seismic Noise: The Earth is highly unstable for the ultra-sensitive GW interferometers. The plates in the Earth's crust constantly move which results in constant vibrations. Such seismic noises disturb the lasers' completely incoherent phases resulting in the appearance of an interference pattern at the PD. Thus, the detector must be completely isolated from the ground through various vibrationally-isolating techniques. The first technique in the seismic isolation system is the suspensions from which the test masses or the end mirrors are hanged. The suspension is a complex system of pendulums and springs comprising of multiple stages. It is composed of 4-stages of pendulum and 3 stages of cantilevered blade springs. The fundamental idea of using pendulums is that if such a system oscillates well above its resonant frequency, the hanging test masses (mirrors) won't move at all. Thus, if the ceiling vibrates horizontally due to seismic activities, the suspension won't allow the test masses to move. So, in principle, the test-masses would be free to move whenever a gravitational wave passes yet secure from seismic

noises. Strong springs are then used to provide vertical isolation. The isolation system also consists of seismometers that constantly measure any seismic activity and instruct the control system installed on the suspension systems to induce the opposite motion to counter the seismic noise.

3. **Thermal Noise:** Thermal noise is basically rooted in the motions of the millions of atoms and molecules making the apparatus. Gas molecules in the arms of the interferometer are prime contributors of thermal noise. Due to Brownian motion, these tiny molecules collide with the mirrors and displace them from their absolutely still position by an infinitesimally small amount. Given the displacement sensitivity needed to detect gravitational waves, this tiny displacement is an enormous source of noise. Thus, to reduce the thermal noise caused by the Brownian motion of gas particles, the interferometer is stored in ultra-high vacuum cavities. This significantly reduces thermal noise introduced by gas particles. However, there are numerous sources of thermal noise. There is also internal friction in the mirrors which can cause noise similar to the gas molecules discussed above. Thus, it

becomes necessary to use mirrors that have low internal friction. This is why mirrors composed of fused silica are useful. Fused silica has a rigid 3-dimensional structure offering low thermal expansion coefficient and low internal friction. It is also known that even if the mirrors are completely isolated from the outside environment, thermal noise still finds its way into the suspension cables. The suspension cable (also made of fused silica), is circular in shape but with variable diameter. This strategy of variable diameter minimizes thermal noise in the cable.

4. **Quantum Noise:** There are a variety of variables that are called conjugates of each other i.e. they have a form provided by the Heisenberg's Uncertainty Principle. One such pair is laser power and radiation pressure. Increasing laser power leads to an improvement in the accuracy of the interferometer in detecting gravitational waves as well as the elimination of photon shot noise. However, when laser power is too high, it heats up the coatings of the mirrors and hence transfers momentum to the mirrors which takes the form of radiation pressure. Thus, we need to compromise one variable for the other. However, very high

uncertainty in the radiation pressure cannot be under-judged. It would lead to the movement of the hanging test masses. Thus, experimentalists are currently employing a method called the 'squeezing light'

3. <https://dcc.ligo.org/public/0072/P1100131/003/CR%20GW%20review%20Saulson%20v3.pdf>

4. <https://arxiv.org/pdf/1411.4547.pdf>

Conclusion

So far we have briefly reviewed the nature of gravitational waves, how they are detected and the practical difficulties in measuring them. However, there are many mechanisms- both theoretical and practical- that cannot be accommodated in such a short review. There are a lot of other noise sources and so do their reduction techniques. Such large scale and highly collaborative experiments as detection of gravitational waves truly incorporate ingenuity from all fields of science and technology. To conclude, we appreciate the success of highly sensitive interferometers currently operating like aLIGO, aVIRGO and GEO 600.

References

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